2023

ADVANCED BUSINESS MATHEMATICS — HONOURS

Paper : DSE-5.1 AH

(Module - II)

Full Marks : 40

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer *any four* questions.

1. (a) Evaluate : $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x+x^2}-1}{x}$.

   (b) The function $f(x) = \dfrac{x^3-8}{x^2-4}$ is undefined at $x = 2$. Redefine the function so as to make it continuous

   at $x = 2$.   5+5

2. (a) Evaluate : $\displaystyle\int_{-1}^{0} \dfrac{1+x}{1-x}\,dx$.

   (b) Evaluate : $\displaystyle\int \dfrac{dx}{(x-3)\sqrt{x+1}}$.   5+5

3. (a) If $f(x) + 2f(-x) = x^2$, show that $f(3) = 3$.

   (b) If $y = 2x^3 + 3x^2 - 36x + 7$, find the values of x for which $\dfrac{dy}{dx} = 0$.   5+5

4. (a) Solve by Cramer's Rule : $2x - y + 3z = -2$, $3x + 2y + 3z = 9$, $x + 2y - z = 9$.

   (b) Show that the matrix $A = \dfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. Hence, find A^{-1}.   5+5

Please Turn Over

5. (a) Evaluate : $\int (2x+1)\sqrt{4x^2 + 4x + 5}\, dx$

(b) If $x = t^2 + 2t$, $y = t^2 - 2t$, find out $\dfrac{d^2 y}{dx^2}$ at $t = 1$.   5+5

6. (a) Without formal expansion prove that $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = x^3 + y^3$.

(b) Prove that $\left(1 + \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}\right)$ is a factor of $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$.   5+5

7. (a) The price per unit $p(x)$ at which a company can sell all that it produces is given by $p(x) = 300 - 4x$ and that cost function is $c(x) = 300 + 52x$, where x is the number of units produced. Find x so that profit is maximum.

(b) Find the area bounded by straight lines $3x - 2y = 6$, $2x + y = 10$ and x axis.   5+5

8. (a) Let A and B be two matrices such that $AB = \begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$. Find out A.

(b) If $P = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix}$, then show that $P^2 = P$ and hence find matrix Q such that $4P^2 - 2P + Q = O$, where O is the zero matrix of order 3.   5+5

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